114_006 Coin-Flipping Etc

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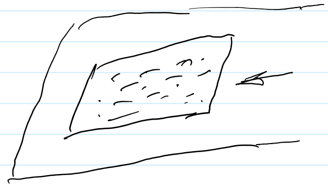
Coin flipping, **coin tossing**, or **heads or tails** is the practice of throwing a coin in the air and checking which side is showing when it lands, in order to choose between two alternatives, heads or tails, sometimes used to resolve a dispute between two parties.
It is a form of sortition which inherently has two possible outcomes.
The party who calls the side that the coin lands on wins.
From <https://en.wikipedia.org/wiki/Coin_flipping>



Dice throwing

Card game - Poker

$\mathcal{A}: PrK_A = x,\ PuK_A = a;$ $\quad\mathcal{B}: PrK_B = y,\ PuK_B = b;$ $\quad\mathcal{E}: PrK_C = z,\ PuK_C = e;$

$b, e;\quad a = g^x \bmod p$ $\quad a, e;\ b = g^y \bmod p;$ $\quad a, b;\ e = ---$

ElGamal encryption

PP=(*p*,*g*) **>> p = 268 435 019**; % 2^28 -1 --> >> int64(2^28-1)
% ans = **268 435 455**
>> g=2;

$m \in \mathbb{Z}_p^* = \{1, 2, 3, ..., p-1\};\ *\ \bmod p$

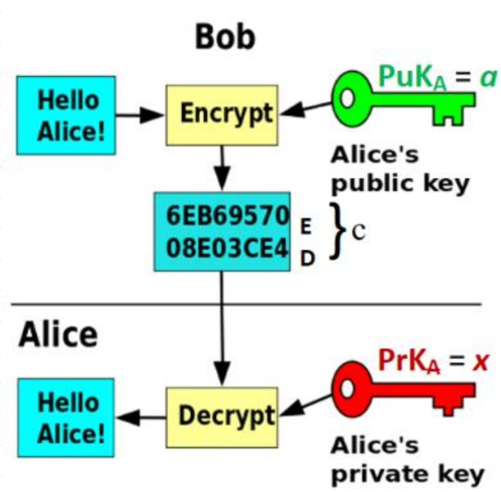
message to be encrypted

$i \leftarrow \mathrm{randi};\quad i \in \mathbb{Z}_{p-1} = \{0, 1, 2, ..., p-2\}$

$c = Enc(a, i, m) =$

$= (E, D) = (\underbrace{m a^i \bmod p}_{E}, \underbrace{g^i \bmod p}_{D})$

$Dec(x, c) = E \cdot D^{-x} \bmod p = \frac{E}{D^x} \bmod p =$



$$Dec(x, c) = E \cdot D^{-x} \bmod p = \frac{E}{D^{x}} \bmod p =$$

$$= \frac{m\, a^{i} \bmod p}{(g^{i})^{x}} = \frac{m\, (\cancel{g^{x}})^{i} \bmod p}{\cancel{g^{ix}}} = m \bmod p = m \quad \text{if } m < p$$

D^{-x} mod p computation using Fermat theorem:
If p is prime, then for any integer a holds $a^{p-1} = 1 \bmod p$.

$$D^{-x} = D^{p-1-x} \bmod p$$

$D^{-x} \bmod p$ computation

a) D^{-1} computation : >> D_m1=mulinv(D, p)

b). D^{-x} computation : $(D^{-1})^{x} = D^{-x}$ >> D_mx=mod_exp(D_m1,x, p)

$\mathcal{A}$: $PrK_A = x$; $PuK_A = a$; $PuK_B = b$;

$m_i \in \{1, 2\}$

Coin flipping scheme: ***Alice*** before coin flipping assigns possible results to variables $m_1=1$ and $m_2=2$.
1. ***Alice*** after coin flipp assigns result m either to $m=1$ or $m=2$.
2. ***Alice*** encrypts $m_1=1$ and $m_2=2$ by her **PuK**=a using random generated numbers $i1$ and $i2$ computing ciphertexts c_{1A} and c_{2A} respectively:

$m_i \in \{1, 2\}$

$i_1, i_2 \leftarrow randi(\mathcal{Z}_{p-1})$

$$\left.\begin{array}{l} c_{1A} = Enc(a, i_1, m_1) = (E_{1A}, D_{1A}) \\ c_{2A} = Enc(a, i_2, m_2) = (E_{2A}, D_{2A}) \end{array}\right\}$$

$$\left.\begin{array}{l} E_{1A} = m_1 \cdot a^{i_1} \bmod p;\ D_{1A} = g^{i_1} \bmod p \\ E_{2A} = m_2 \cdot a^{i_2} \bmod p;\ D_{2A} = g^{i_2} \bmod p \end{array}\right\} \quad \xrightarrow{c_{1A},\, c_{2A}} \quad \mathcal{B}$$

$\mathcal{B}$: $PrK_B = y$; $PuK_B = b$.

$\mathcal{A}$:

$c_{2A} \leftarrow rand\{c_{1A}, c_{2A}\}$; $c_{iA} = c_{2A}$

$i_3 \leftarrow randi(\mathcal{Z}_{p-1})$

$$Enc(b, i_3, E_{2A}) = (E_{2AB}, D_{2AB}) = c_{2AB}$$

$$= (E_{2A} \cdot b^{i_3} \bmod p,\ g^{i_3} \bmod p)$$

$$= (\underbrace{\underbrace{E_{2A} \cdot b^{i_3} \bmod p}_{E_{2AB}}, \underbrace{g^{i_3} \bmod p}_{D_{2AB}}}_{})$$

$$\xleftarrow{E_{2AB}} C_{2AB}$$

$$Dec(x, C_{2AB}) = \frac{E_{2AB}}{(D_{2A})^x} = E_{2AB} \cdot (D_{2A})^{-x} =$$

$$= \frac{E_{2A} \cdot b^{i_3}}{(g^{i_2})^x} = \frac{m_2 \cdot a^{i_2} \cdot b^{i_3}}{g^{i_2 x}} =$$

$$= \frac{m_2 \cdot a^{i_2} \cdot b^{i_3}}{g^{i_2 x}} = \frac{m_2 \cdot \cancel{g^{x i_2}} \cdot b^{i_3}}{\cancel{g^{i_2 x}}} =$$

$$= m_2 \cdot b^{i_3} = E_{2ABA} \xrightarrow{E_{2ABA}} B: C_{2ABA} = (E_{2ABA}, D_{2AB})$$

1 Let B guessed that A tossed C_{2A}

$$Dec(y, C_{2ABA}) = E_{2ABA} \cdot (D_{2AB})^{-y} =$$

$$= \frac{E_{2ABA}}{(D_{2AB})^y} = \frac{m_2 \cdot b^{i_3}}{(g^{i_3})^y} = \frac{m_2 \cdot (g^y)^{i_3}}{g^{i_3 y}} =$$

$$= \frac{m_2 \cdot \cancel{g^{y i_3}}}{\cancel{g^{i_3 y}}} = m_2$$

$$\xleftarrow{m_2,\ i_3}$$

$$m = E_{2ABA} \cdot (b)^{-i_3} \bmod p =$$

$$m_2 \cdot b^{i_3} \cdot b^{-i_3} = m_2 \cdot b^{i_3 - i_3} =$$

$$= m_2 \cdot b^0 = m_2 \cdot 1 = m_2$$

$$\xrightarrow{i_2} B: C_{2A} = (E_{2A}, D_{2A})$$

$$E_{2A} \cdot a^{-i_2} \bmod p =$$

$$= m_2 \cdot a^{i_2} \cdot a^{-i_2} \bmod p =$$

$= m_2 \cdot a^{i_2 - i_2} = m_2 \cdot a^0 = m_2$

(2) Let $\mathcal{B}$ choosed that $\mathcal{A}$ tossed $c_{1A} = (m_1 \cdot a^{i_1}, g^{i_1})$

$\mathcal{B}$ did not guess the toss.

$i_3 \leftarrow \text{randi}(\mathbb{Z}_{p-1})$

$Enc(b, i_3, E_{1A}) = (E_{1AB}, D_{1AB}) = c_{1AB}$

$$= \underbrace{\underbrace{(E_{1A} \cdot b^{i_3} \bmod p}_{E_{1AB}}, \underbrace{g^{i_3} \bmod p)}_{D_{1AB}}}_{c_{1AB}}$$

$\xleftarrow{\quad c_{1AB} \quad}$

$$Dec(x, c_{2AB}) = \frac{E_{1AB}}{(D_{2A})^x} =$$

$$= \frac{E_{1A} \cdot b^{i_3}}{(g^{i_2})^x} = \frac{m_1 \cdot a^{i_1} \cdot b^{i_3}}{g^{i_2 x}} =$$

$$= \frac{m_1 \cdot a^{i_1} \cdot b^{i_3}}{g^{i_2 x}} = \frac{m_1 \cdot g^{x i_1} \cdot b^{i_3}}{g^{i_2 x}} =$$

$= E_{1ABA}$ $\xrightarrow{\quad E_{1ABA} \quad}$

$\mathcal{B}$:

$$Dec(y, c_{1ABA}) =$$

$$= \frac{E_{1ABA}}{(D_{2AB})^y} = m'$$

$E_{1ABA} \cdot (b)^{-i_3} \bmod p = m''$ $\xleftarrow{\quad m', i_3 \quad}$

$\xrightarrow{\quad i_2 \quad}$ $\mathcal{B}$: $Dec(y, m'')$

$E_{1A} \cdot a^{-i_2} \bmod p = u \notin \{1, 2\}$

If random generated number i in ElGamal encryption is revealed then ciphertext canbe decrypted.

$Enc(a, i, m) = (E, D) = (m \cdot a^i \bmod p, g^i \bmod p) = c$

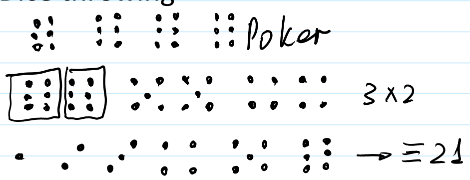
Decryption without knowledge $PrK = x$ but having i:

$Enc(\alpha, i, m) = (E, D) = (m \cdot \alpha^i \bmod p,\ g^i \bmod p) = c$

Decryption without knowledge PrK = x but having i:

$E \cdot \alpha^{-1} \bmod p = m\cancel{\alpha^i}\cancel{\alpha^{-i}} \bmod p = m \bmod p = m.$

Dice throwing



$m \in \{1, 2, 3, 4, 5, 6\}$

$r_1 \leftarrow randi, \ldots, r_6 \leftarrow randi$

$c_i = Enc(\alpha, r_i, m_i), i = \overline{1,6}.$

$c_1 \equiv 1;\ c_2 \equiv 2;\ c_3 \equiv 3; \ldots c_6 \equiv 6.$ → B: $c_i \leftarrow rand\{c_i\}$

→ C: $c_i = c_6$

$c_{ij} = Enc(\alpha, r_{ij}, m_{ij})$ $c_{ij} \leftarrow rand\{c_{ij}\}$

$i = \overline{1,6}$ kauliuko reikšmės

$j = \overline{1,6}$ kauliuko numeris

Card game - Poker

52 kortos & 4 mostis

1 kortos šifrav.

$c_i = Enc(\alpha, r_i, m_i);\ i = \overline{1,4}.$

$c_{ij} = Enc(\alpha, r_{ij}, m_{ij});\ j = \overline{1,52}.$